examples from calc 1

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## Tuples

* simple; **non-mutable** version of lists
* faster, safer
* can be expressed as x, y, z (or (x,y,z), probably clearer)
* empty tuple: ()
* tuple with one element: (x,)
* can do many of the same things as with lists

x = (1,4,"a",3)
print(x[1]) ## indexing

## 4

print(x[2:]) ## slicing

## ('a', 3)

print(x+(3,)) ## appending

## (1, 4, 'a', 3, 3)

print(x[:2] + (3,) + x[2:]) ## insertion in the middle

## (1, 4, 3, 'a', 3)

x.index(4) ## indexing

## 1

"z" in x ## looking for stuff

## False

x.count(4) ## count

## 1

* you *can’t* modify the existing tuple at all (deletion, modification)
* unpacking: x,y,z = t
* swapping: (a,b) = (b,a)
* useful as the return value of functions; safe, and can be unpacked
* convert to/from lists (tuple(), list())

x = (1,2,3)
def modify(x):
 y = list(x)
 y[0] = "a"
 return(tuple(y))

print(modify(x))

## ('a', 2, 3)

print(x)

## (1, 2, 3)

## reminders/clarifications

* parentheses (()) vs square brackets ([])
* **square brackets**
	+ indexing (lists or strings or tuples): x[5]
	+ slicing (lists or strings or tuples): x[5:7]
	+ defining lists: [1,2,3]
* **parentheses**
	+ order of operations: (1+2)\*3, a and (not b or c)
	+ calling functions: len(x), range(5), print("hello")
	+ calling methods: x.sort(), x.append(4)
	+ defining functions: def f(x1,x2,x3):
	+ returning values from functions: return(x) (\*)
	+ defining tuples: (), (1,), (2,3) (\*)

“*" actually (mostly)* optional\*: see [here](https://stackoverflow.com/questions/4978567/should-a-return-statement-have-parentheses)

## Root-finding methods

* Assume that $f(x)$ is a continuous function on the real numbers.
* Suppose that $a<b$
* Suppose *endpoints are of opposite signs*:
$f(a)<0$ and $f(b)>0$ **or** $f(b)<0$ and $f(a)>0$
* (or $f(a)⋅f(b)<0$)
* By the Intermediate Value Theorem, there is some number $c$ between $a$ and $b$ with $f(c)=0$; this is called a **root** of the function $f$

We will use three methods (Grid, Bisection, and Newton’s method) to approximate such a number $c$. (There may be more than one root of $f$ in the interval between $a$ and $b$.)

## Example

* We’ll use $exp(x)−x−3/2$ as an example
* impossible to do analytically!
* value at 0 = -3/2
* value at 1 = $exp(1)−1−3/2≈2.78−2.5=0.28$

## Grid Method

* Break the interval $[a,b]$ into $n$ subintervals of equal sizes, having endpoints

$$x\_{0}=a,x\_{1},…,x\_{n−1},x\_{n}=b .$$

* Compute $f(x\_{0}),f(x\_{1}),…,f(x\_{n})$
* Find the index $i$ such that $f(x\_{i})$ is closest to 0 and use this to approximate a root of $f$ in the interval $[a,b]$.
* **Project:** Create a function grid\_search(f, a, b, n) that implements the grid method.

## Bisection Method

* Bisect the interval $[a,b]$ into two equal subintervals $[a,m]$, $[m,b]$, where $m=(a+b)/2$.
* If $f(a)$ and $f(m)$ have opposite signs, then there will be a root in $[a,m]$. Otherwise, there will be a root in $[m,b]$.
* Bisect this subinterval ($[a,m]$ in the former case, $[m,b]$ in the latter), and continue bisecting until the subinterval is small.
* A root of $f$ will be located in this small subinterval.
* **Project:** Create a function bisect(f, a, b, tol) that approximates a root of f in the interval [a, b] with an error of at most tol.

## Newton’s Method

* Suppose we know the derivative (*gradient*) $df/dx=f′(x)$ as well as $f(x)$
* For a given starting value $x\_{0}$, guess the position of the root according to $x\_{1}=f(x\_{0})−\frac{x\_{0}}{f′(x\_{0})}$.
* Repeat until we are within tolerance of the root ($|f(x)|$ is small
* **Project:** Create a function newton(f, grad, x0, tol, nmax) that approximates a root of f with an error of at most tol, taking no more than nmax steps.