Name _____

Student Number _____

MATH 1MP3

DAY CLASS
DURATION OF EXAMINATION: 2 Hours — Benjamin Bolker
MCMASTER UNIVERSITY FINAL EXAMINATION — April 2016

THIS EXAMINATION PAPER INCLUDES 5 PAGES AND 9 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions:

- please *circle your family name* above
- **no** external aids (notes, calculator, etc.)
- This paper must be returned with your answers.

---

1. Suppose you are given a dictionary of the form

```
d = {'joe':("male",25), 'fred':("male",39), 'susan':("female",20)}
```

where each key is a name and each value is a tuple containing the sex and age of that individual.

   a. (3 points) write code to count the number of males between the age of 20 and 30 (inclusive) ... (in this example, the correct answer would be 1)
   b. (3 points) generalizing your previous answer, write a function `count_dict(d,sex,age_lwr,age_upr)` that returns the number of individuals of a specified sex between the age limits. If `sex` is neither `"male"` nor `"female"` it should raise a `ValueError`.
   c. (4 points) suppose now that you have the following type of data instead, where the names are defined in a separate dictionary

```
d = {'joe':25, 'fred':39, 'susan':20}
names = {'joe':"male", 'fred':"male", 'susan':"female"}
```

write a function `count_dict2(d,name_dict,sex,age_lwr,age_upr)` that handles this kind of data to solve the same problem defined above.

solution

a.

```
count = 0
for k in d:
    sex, age = d[k]
    if sex=="male" and 20<=age<=30:
         count += 1
```

b.

```
def count_dict(d,sex,age_lwr,age_upr):
   count = 0
   for k in d:
      cur_sex, cur_age = d[k]
      if not cur_sex in ("male", "female"):
          raise ValueError
      if cur_sex==sex and age_lwr<=cur_age<=age_upr:
          count += 1
   return count
```

c.

```
def count_dict2(d,name_dict,sex,age_lwr,age_upr):
    count = 0
    for k in d:
        cur_age = d[k]
        cur_sex = name_dict[k] # assume key is in name_dict too (not specified)
        if not cur_sex in ("male", "female"):
            raise ValueError
        if cur_sex==sex and age_lwr<=cur_age<=age_upr:
            count += 1
    return count
```

Rubric:

- several people asked what to call the dictionary given in the example. Anything reasonable (`d`, `dict`, ...) is fine.
- there is some possibility for confusion about whether the `ValueError` needs to be raised when the *argument* `sex` has an invalid value, or when one of the elements in the *dictionary* has an invalid value. The first is what I meant, and more sensible, but either is OK. The `ValueError` may, but need not have, an associated error message.
- -1 for a minor logic flaw; -2 for a major logic flaw (i.e., 1 point for writing *something* reasonable)

2. (6 points) The Bessel function can be defined as

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha}$$

(Wikipedia)

The factorial (`m!`) and Gamma ($\Gamma(.)$) functions can be imported from `scipy` via

```
from scipy.special import gamma,factorial
```

assuming that these functions have already been imported, write a function `besselJ(x,alpha,k=4)` that returns the (approximation to the) Bessel function computed by summing the terms in the series up to *and including* the k^{th} term (i.e. $\sum_{m=0}^{k}$). (You can assume that the input is legal, i.e. that `x` is a non-negative floating point number, `alpha` is a floating point number, and `k` is an integer.)

solution

```
def besselJ(x,alpha,k=4):
   result = 0
   for m in range(k+1):
      result += (-1)**m/(factorial(m)*gamma(m+alpha+1))*(x/2)**(2*m+alpha)
   return result
```

tangential comment: this computation would be better, since it does a lot of the computation on the log scale (avoiding floating-point overflow)

```
from scipy.special import gammaln
import math
def logfactorial(x):
   return gammaln(x+1)
def besselJ(x,alpha,k=4):
   result = 0
   for m in range(k+1):
      result += (-1)**m*math.exp(-logfactorial(m)-gammaln(m+alpha+1)+(2*m+alpha)*math.log(x/2))
   return result
```

rubric:

- -1 for getting the range wrong (I was pretty careful to be explicit)
- -0.5 (each) for using the exclamation point instead of `factorial` or the symbol Γ or the capitalized word `Gamma` instead of the corresponding functions
- -1 for other minor logic errors

3. There is something wrong with each of the following examples: they "should" produce a `True` value, but they don't (they produce either a non-`True` value or an error). State what value/error they produce and give a *short* (one-sentence) explanation what has gone wrong. (2 points each)

a. check that $\left(\sqrt{2}\right)^2 = 2$:

```
import numpy as np
np.sqrt(2)**2==2
```

```
## False
```

rubric: say *something* about "floating point error", "floating point imprecision", "numerical precision", etc.

b. list reversal:

```
def rev(x):
   x.reverse()
   return(x)

L = [0,1,2,3]
L_rev = rev(L)
L[1] == 1
```

```
## False
```

rubric: say *something* sensible about mutability, or the equivalent in words ("`L` and `L_rev` are (pointing at) the same object")

c. extract the third element of a list:

```
a = [1,2,3]
a[3] == 3
```

rubric: say *something* about a range error

d. compute $\sum_{i=0}^{3} i^2$:

```
for i in range(4):
   k = 0
   k += i**2
k == 14
```

```
## False
```

rubric: say something that indicates that `k=0` should be outside the loop (not necessary to say that the result will be 9)

4. Collatz conjecture

a. (6 points) Write a function `def collatz(n,itmax=1000)` that, for any given value of `n`,
   - if `n` is even, divide it by 2
   - if `n` is odd, multiply it by 3 and add 1

and continues these steps until more than `itmax` steps have been taken *or* `n` is equal to 1. The function should return the total number of times through the cycle. For example, for `collatz(5)`, the sequence would be 5, 16, 8, 4, 2, 1 and the function would return 5. For `collatz(6)` the sequence would be 6, 3, 10, 5, 16, 8, 4, 2, 1 and the function would return 8.

rubric: I hope people don't get confused and return the list instead of the length of the list.

- -0.5 for doing the problem correctly but returning the list
- -0.5 for off-by-one error in counting the length
- -1 for ignoring `itmax`, but don't worry about the distinction between `<itmax` and `<= itmax`
- -1 for minor logic errors

b. (3 points) Using this function, write Python code that computes the number of steps required for each value between 1 and 10000 (inclusive) and saves the results in a `numpy` array (plotting the resulting array would produce the following picture ... which is, however, completely irrelevant for the purposes of the exam)

rubric: I intended a 1-D array, but a 2-D array with the indices in it would be OK too. `for` loops are expected. List comprehensions are too clever, but would be acceptable.

```
def collatz(n,itmax=1000):
   it = 0
   while n>1 and it<=itmax:
      if n % 2 == 0:
          n = n / 2
      else: n = 3*n+1
      it += 1
   return(it)
import numpy as np
n = 10000
```

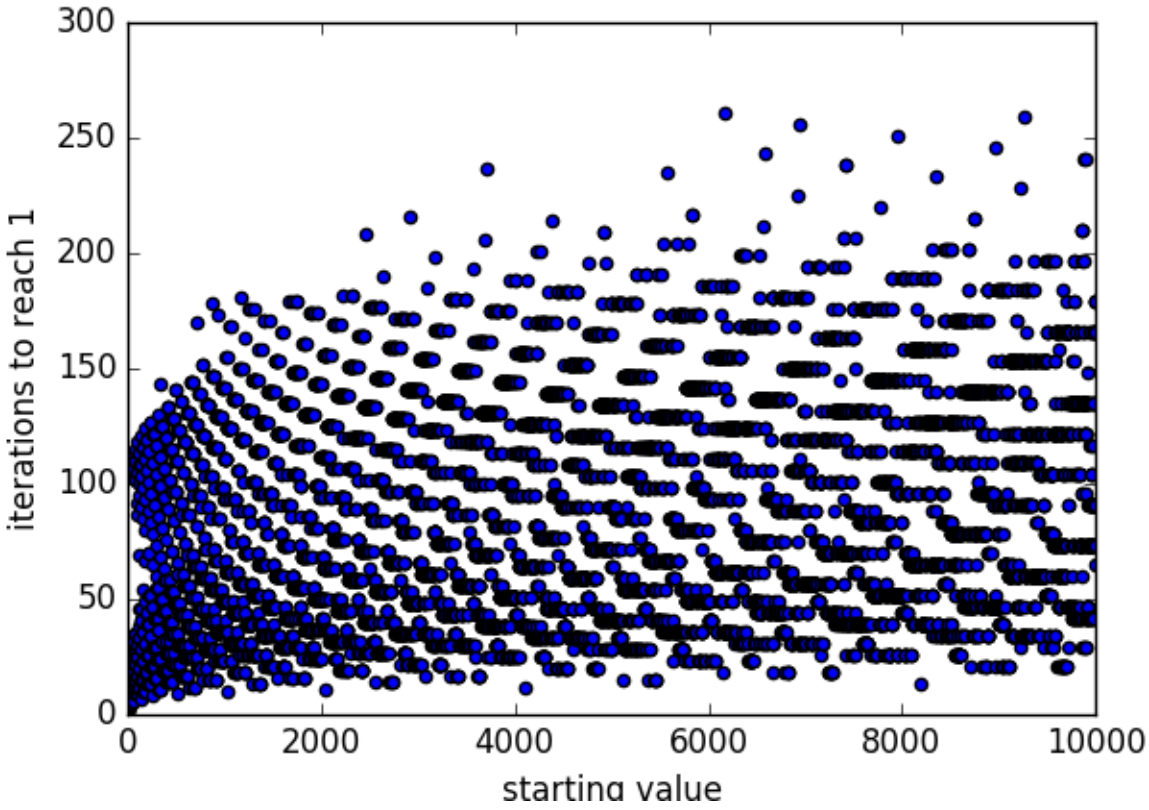
```
r = range(1,n)
cvals = np.array([collatz(n) for n in r])
## Plotting code: **not required** as part of the answer
import matplotlib.pyplot as plt
fig, ax = plt.subplots()
ax.scatter(r,cvals)
ax.set_xlim(0,n)
```

```
## (0, 10000)
```

```
ax.set_ylim(0,300)
```

```
## (0, 300)
```

```
ax.set_xlabel("starting value")
ax.set_ylabel("iterations to reach 1")
fig.savefig("collatz.png")
```



5. (6 points) The function `os.listdir()` returns a list of the names of files found in a directory. Suppose that `L` is the result of this command, and that every file in the directory contains a single column of numbers, and that every file has the same number of rows. Write a program that reads each file and combines them into a single `numpy` array *of floats*. Keep in mind that:

- if `fn` is a file name, `open(fn)` opens the file;
- if `f` is an open file, `f.read().split()` will read the entire file and split it on whitespace, returning a list *of characters*:
- `numpy.array` has a `dtype` argument that will convert its argument to the specified type

For example, if there were three files in the directory: `a.txt`, `b.txt`, and `other_file.txt`,

```
a.txt          b.txt          other_file.txt
------         -------        --------------
1              17             4
2              18             5
3              150            6
```

then the result would be

```
## [[  1.  17.   4.]
##  [  2.  18.   5.]
##  [  3. 150.   6.]]
```

rubric: people might do this by setting up an empty array of the appropriate dimension and setting the columns, or by appending results to a list of lists and then turning it into an array and then transposing it.

- 3 points for doing *something* sensible
- -1 point for getting the transpose of the correct answer.

6. (5 points) Draw an approximation of the picture that the following code produces. Include x- and y-axis limits.

```
import numpy as np
import matplotlib.pyplot as plt
x = np.arange(10)
fig, ax = plt.subplots() ## open a figure containing a single axis
ax.plot(x,x**2)
ax.scatter(x,-np.sqrt(x))
fig.show()
```

rubric:

- -1 for not distinguishing between `plot` (solid line by default) and `scatter` (points by default)
- -1 for fundamental mistakes about what `-np.sqrt(x)`, `x**2` look like
- -0.5 for small mistakes about the range (should be approx. x from 0 to 9, y from -3 to 100) (overall: errors don't stack)

7. (3 points for each item) Given a two-dimensional `numpy` array `a`, write a single line of code **using slicing or ranges** to extract various components. As an example, suppose `a` is of the form

```
 1   2   3 ...   4   5
17  21  18 ...  90  91
 4   6   9 ...   8   7
... ... ... ... ... ...
12  17  18 ...  21  22
 2   1   7 ...   3   4
 1   8   9 ...   6   4
```

(where `...` stands for some number of omitted rows/columns)

a. the element in the first row, second column (2 in the example)
b. the third row ([4 6 9 ... 8 7] in the example)
c. the last column ([5 91 7 ... 22 4 4] in the example)
d. the last three elements in the last column ([22 4 4] in the example)

rubric: these are pretty much all or nothing. -0.5 per question for small notational mistakes that don't affect the basic logic (e.g. indexing as `a[rows][columns]` rather than `a[rows,columns]`, semicolons vs commas)

8. (3 points for each item) Suppose the file `weather.csv` looks like this:

```
year,month,day,time,temp,wind,wind_dir,precip,precip_type
2014,   01, 01,0800,  -3,   1,      NW,     0,          *
2014,   01, 01,0900,  -2,   0,       *,     0,          *
2014,   01, 01,1000,   0,   0,       *,     2,       snow
2014,   12, 31,1100, -18,   0,       *,     1,       snow
```

Now we run the following `pandas` code:

```
import pandas as pd
dd = pd.read_csv("weather.csv",na_values=["*"])
```

a. what is the value of `dd.loc[2,"temp"]`?
b. what is the value of `dd.iloc[1,6]`? What does this mean?
c. what are the results of running

```
dd2 = dd[(dd.temp<0) & (dd.precip>0)]
print(dd2.precip_type)
```

?

rubric:

- -0.5 mistakes in indexing (e.g. forgetting to count from 0)
- -0.5 for counting the header row as row 0
- -1 for not understanding the difference between `loc` and `iloc`
- -0.5 for reporting the second answer as `*` – best answer is `nan`, "a missing value", but only lose -0.25 for something other than `nan` (e.g. 2.75 for "a mssing value")
- last answer should just be "snow"; this is pretty much all-or-nothing, but 1 point for writing *something* sensible

9. Extra credit (3 points)

What is wrong with this code? Why doesn't it return `True`, and what does it do instead?

```
def foo(x):
   return(x.sort())
a = [1,4,9,2]
b = foo(a)
b[3] == 9
```

The End