generalized linear mixed models

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(Generalized) linear mixed models

(G)LMMs: a statistical modeling framework incorporating:

- combinations of categorical and continuous predictors, and interactions
- (some) non-Normal responses (e.g. binomial, Poisson, and extensions)
- (some) nonlinearity (e.g. logistic, exponential, hyperbolic)
- non-independent (grouped) data

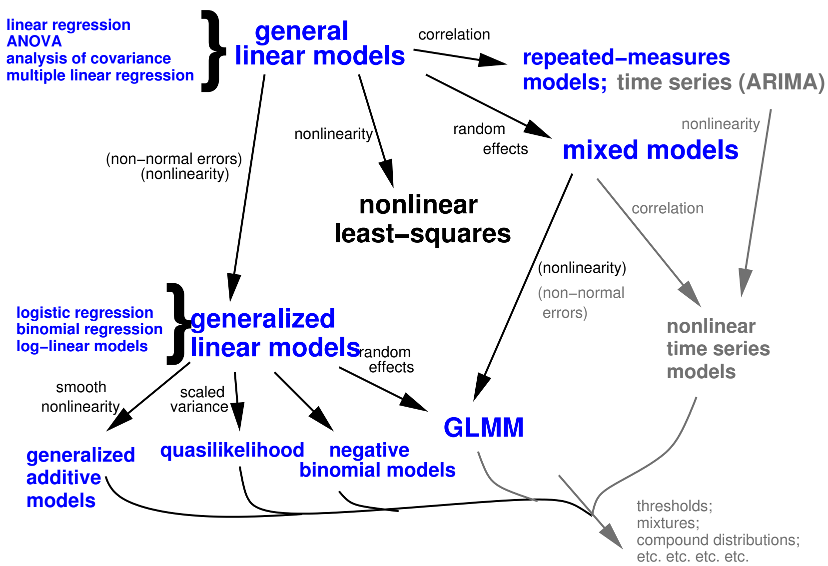
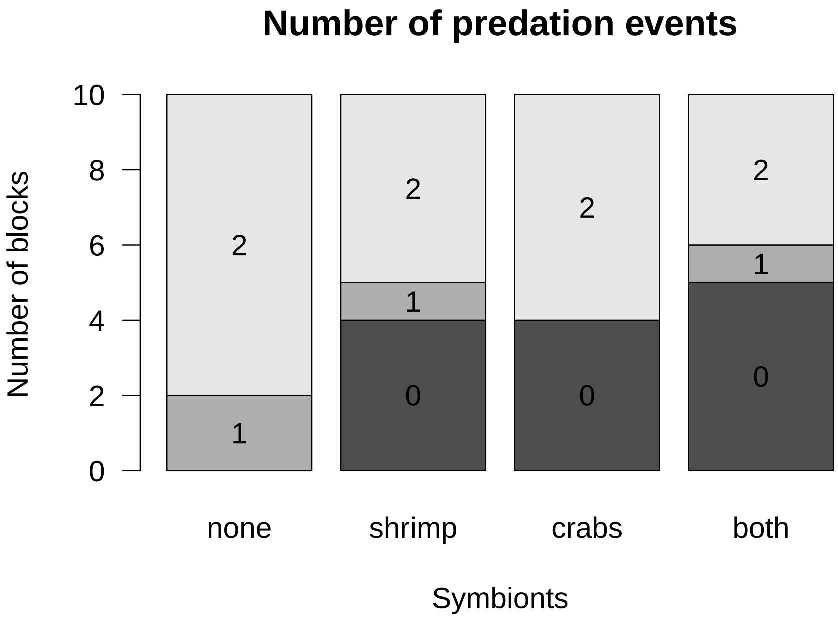
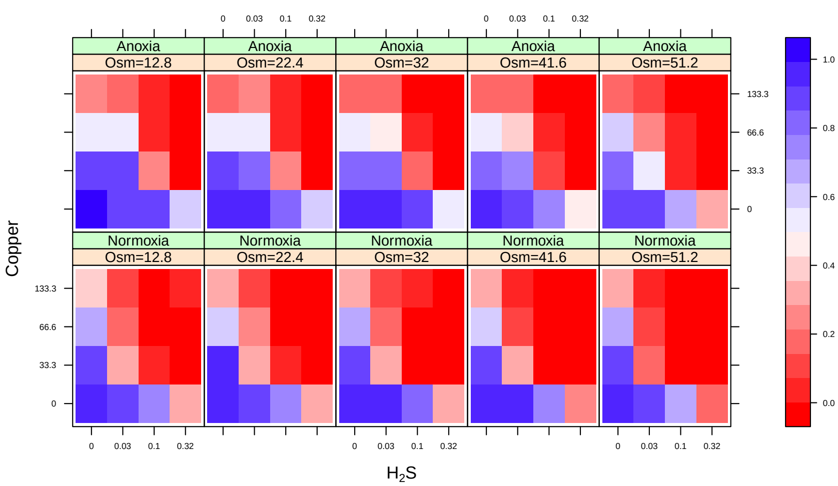


Figure 1: image

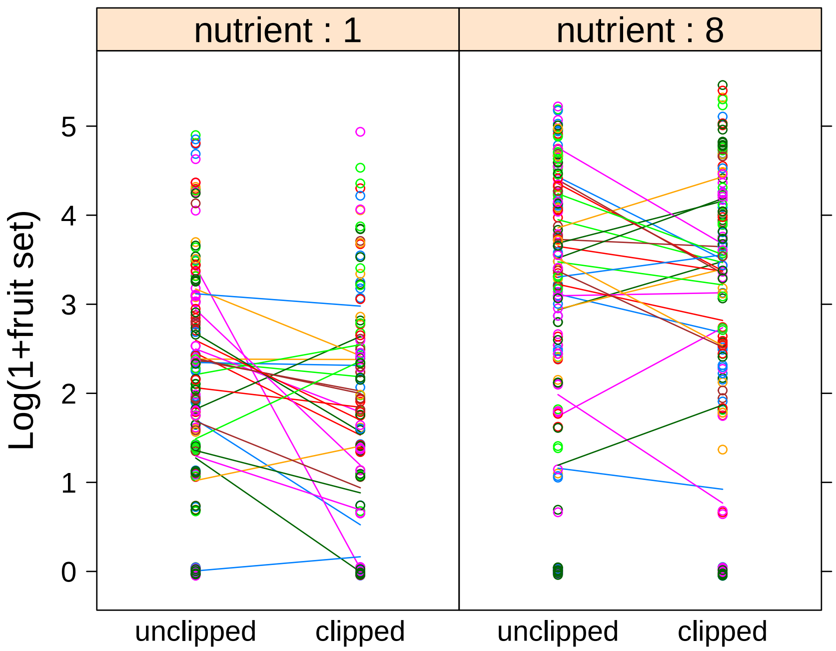
Coral protection from seastars (Culcita) *by symbionts (McKeon et al. 2012)*



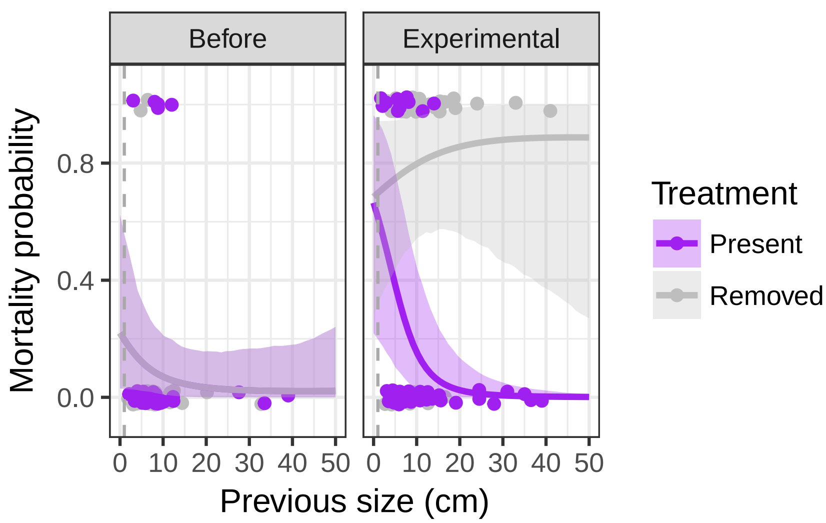
Environmental stress: Glycera *cell survival (D. Julian unpubl.)*



Arabidopsis *response to fertilization & herbivory (Banta, Stevens, and Pigliucci 2010)*



Coral demography (J.-S. White unpubl.)



Technical definition

$$
\begin{aligned}
\underbrace{Y_i}_{\text{response}} &\sim \overbrace{\text{Distr}}^{\substack{\text{conditional}\\\text{distribution}}} \left(\underbrace{g^{-1}(\eta_i)}_{\substack{\text{inverse}\\\text{link}\\\text{function}}}, \underbrace{\phi}_{\substack{\text{scale}\\\text{parameter}}}\right) \\
\underbrace{\boldsymbol{\eta}}_{\substack{\text{linear}\\\text{predictor}}} &= \underbrace{\boldsymbol{X\beta}}_{\substack{\text{fixed}\\\text{effects}}} + \underbrace{\boldsymbol{Zb}}_{\substack{\text{random}\\\text{effects}}} \\
\underbrace{\boldsymbol{b}}_{\substack{\text{conditional}\\\text{modes}}} &\sim \text{MVN}\left(\boldsymbol{0}, \underbrace{\Sigma(\boldsymbol{\theta})}_{\substack{\text{variance-}\\\text{covariance}\\\text{matrix}}}\right)
\end{aligned}
$$

What are random effects?

A method for ...

- accounting for among-individual, within-block correlation
- compromising between
  complete pooling (no among-block variance)
  and *fixed effects* (large among-block variance)
- handling levels selected at random from a larger population
- sharing information among levels (`*shrinkage estimation*`)
- estimating variability among levels
- allowing predictions for unmeasured levels

Random-effect myths

- levels of random effects must always be sampled at random
- a complete sample cannot be treated as a random effect
- random effects are always a *nuisance variable*
- nothing can be said about the predictions of a random effect
- you should always use a random effect no matter how few levels you have

Use a random effect if:

(from B. M. Bolker (2015))

- don't want to test hypotheses about differences between responses at particular levels of the grouping variable;
- do want to quantify the variability among levels of the grouping variable;

- do want to make predictions about unobserved levels of the grouping variable;
- do want to combine information across levels of the grouping variable;
- have variation in information per level (number of samples or noisiness);
- have levels that are randomly sampled from/representative of a larger population;
- have a categorical predictor that is a nuisance variable (i.e., it is not of direct interest, but should be controlled for).

See also Crawley (2002); Gelman (2005)

If you have sampled fewer than five levels of the grouping variable, you should strongly consider treating it as a fixed effect even if one or more of the criteria above apply.

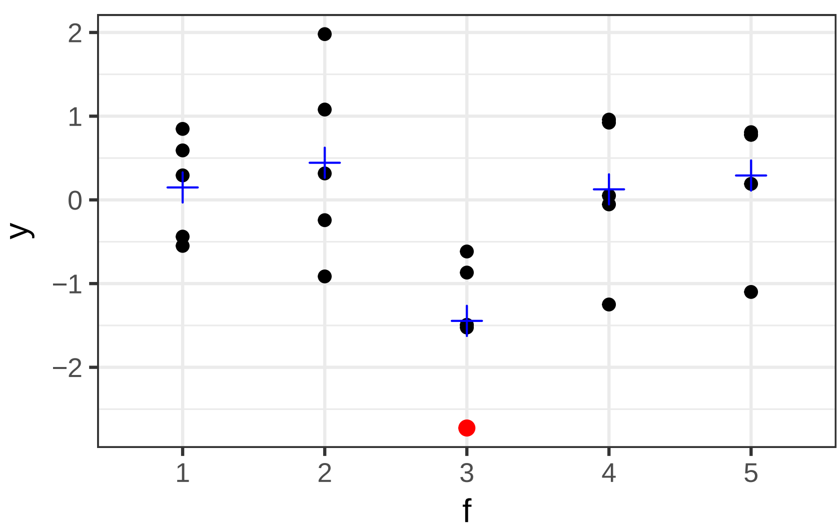
Estimation

Overview

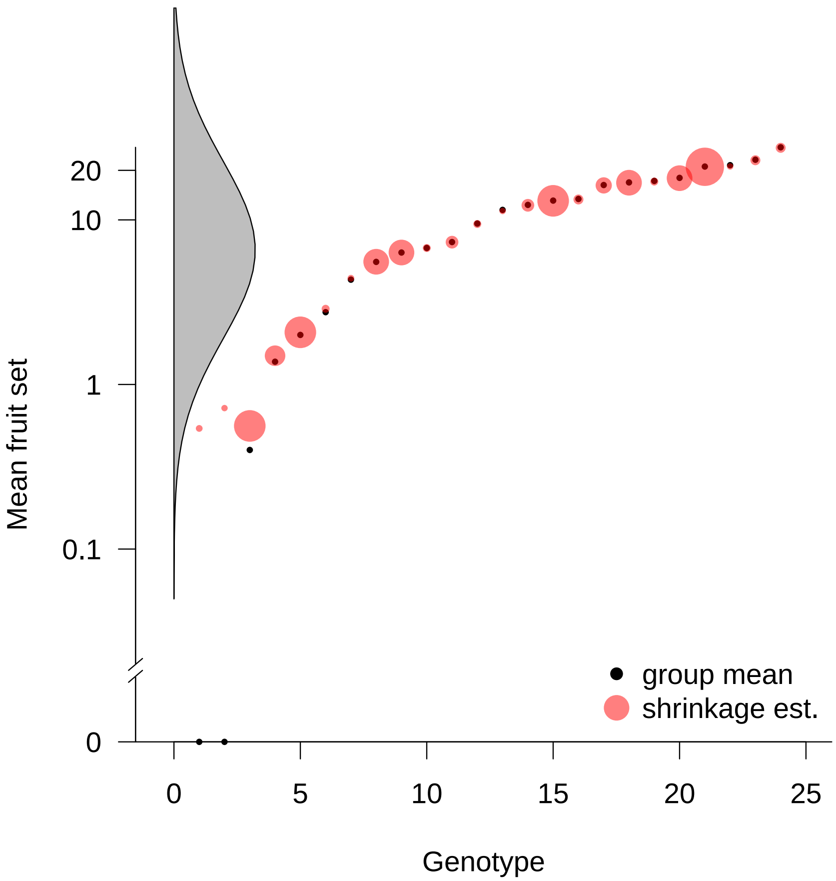
Maximum likelihood estimation

- Best fit is a compromise between two components (consistency of data with fixed effects and conditional modes; consistency of random effect with RE distribution)

- Goodness-of-fit `*integrates*` over conditional modes



Shrinkage: Arabidopsis *conditional modes*

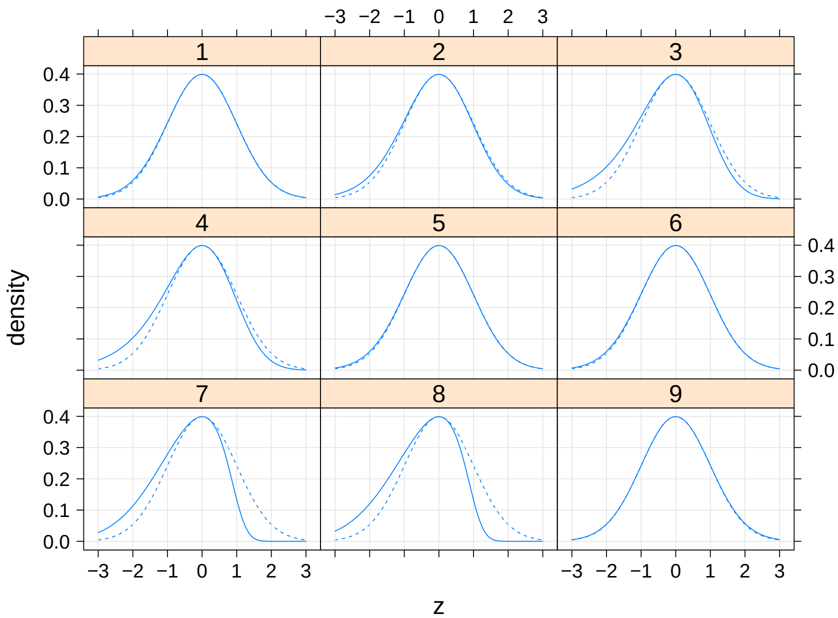


Methods

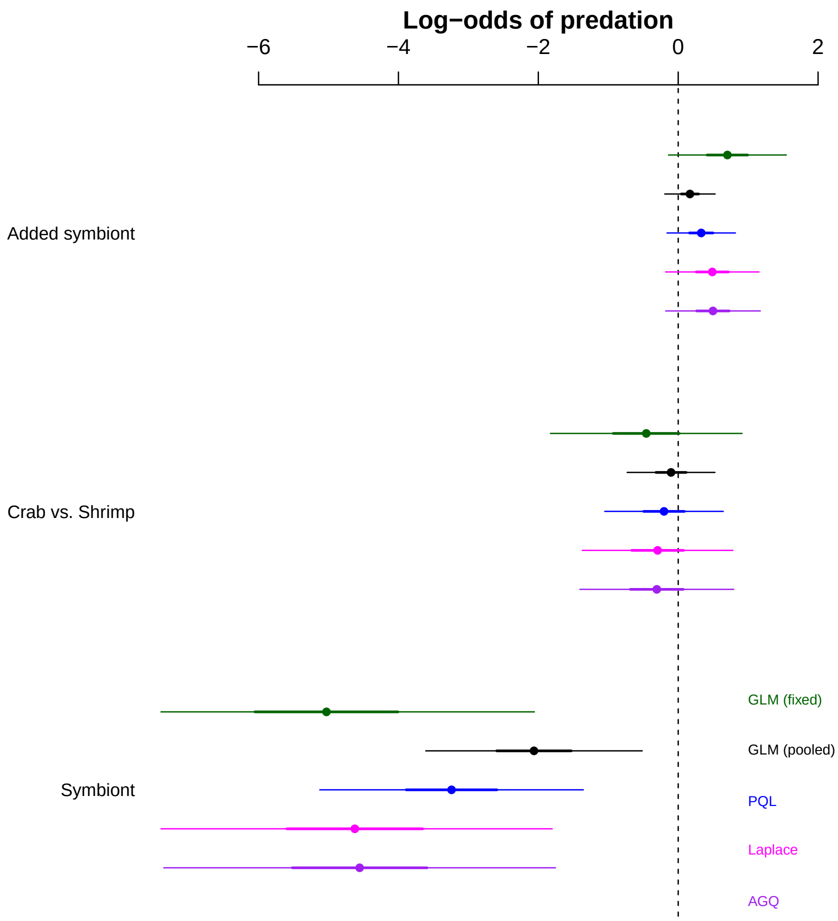
Estimation methods

- deterministic
  - various approximate integrals (Breslow 2004)
  - penalized quasi-likelihood, Laplace, Gauss-Hermite quadrature, . . . (Biswas 2015);
    best methods needed for large variance, small clusters
  - flexibility and speed vs. accuracy
- stochastic
- stochastic (Monte Carlo): frequentist and Bayesian
  - (Booth and Hobert 1999; Sung and Geyer 2007; Ponciano et al. 2009)
  - usually slower but flexible and accurate

Laplace-approximation diagnostics



Estimation: Culcita *(McKeon et al. 2012)*



Inference

Wald tests

- typical results of `summary`
- exact for ANOVA, regression: approximation for GLM(M)s
- fast
- approximation is sometimes awful (Hauck-Donner effect)

Likelihood ratio tests

- better than Wald, but still have two problems:
  - "denominator degrees of freedom" (when estimating scale)
  - for GLMMs, distributions are approximate anyway (Bartlett corrections)
  - Kenward-Roger correction? (Stroup 2014)

- Profile confidence intervals: expensive/fragile

p-values choices?

- guess from classic design (R code)
- conservative: take minimum number of groups - 1
- Satterthwaite/Kenward-Roger (`lmerTest`, LMMs only)
- parametric bootstrap (`pbkrtest`)

Parametric bootstrapping

- fit null model to data
- simulate "data" from null model
- fit null and working model, compute likelihood difference
- repeat to estimate null distribution
- should be OK but ??? not well tested
  (assumes estimated parameters are "sufficiently" good)

Bayesian inference

- If we have a good sample from the posterior distribution (Markov chains have converged etc. etc.) we get most of the inferences we want for free by summarizing the marginal posteriors
- `*post hoc*` Bayesian methods: use deterministic/frequentist methods to find the maximum, then sample around it

Culcita *confidence intervals*

formula formats

- `fixed`: fixed-effect formula
- `random`: random-effect formula (in `lme4`, combined with fixed)
  - generally `x|g` (term|grouping variable)
  - simplest: `1|g`, single intercept term
  - nested: `1|g1/g2`
  - random-slopes: `r|g`
  - independent terms: `(1|g)+(x+0|g)` or `(x||g)`
- `lme`: `weights`, `correlation` for heteroscedasticity and residual correlation
- `MCMCglmm`: options for variance structure

Challenges & open questions

On beyond `lme4`

- `glmmTMB`: zero-inflated and other distributions

- `brms`,`rstanarm`: interfaces to Stan
- `INLA`: spatial and temporal correlations

On beyond R

- Julia: MixedModels package
- SAS: PROC MIXED, NLMIXED
- AS-REML
- Stata (GLLAMM, xtmelogit)
- AD Model Builder; Template Model Builder
- HLM, MLWiN
- JAGS, Stan, rethinking package

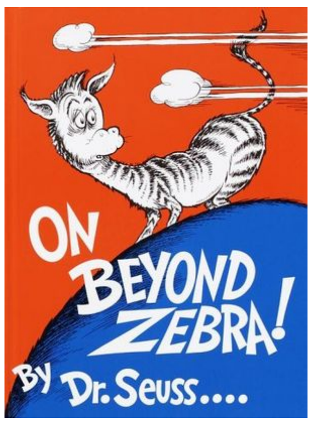


Figure 2: image

Challenges

- Small clusters: need AGQ/MCMC
- Small numbers of clusters: need finite-size corrections (KR/PB/MCMC)
- Small data sets: issues with *singular* fits
  (Barr et al. 2013) vs. (Bates et al. 2015)
- Big data: speed!
- Model diagnosis
- Confidence intervals accounting for uncertainty in variances

See also: `https://rawgit.com/bbolker/mixedmodels-misc/master/ecostats_chap.html` `https://groups.nceas.ucsb.edu/non-linear-modeling/projects`

Spatial and temporal correlations

- Sometimes blocking takes care of non-independence . . .
- but sometimes there is temporal or spatial correlation *within* blocks
- . . . also phylogenetic . . . (Ives and Zhu 2006)

- "G-side" vs. "R-side" effects
- tricky to implement for GLMMs, but new possibilities on the horizon (Rue, Martino, and Chopin 2009; Rousset and Ferdy 2014); https://github.com/stevencarlislewalker/lme4ord

Next steps

- Complex random effects:
  regularization, model selection, penalized methods (lasso/fence)
- Flexible correlation and variance structures
- Flexible/nonparametric random effects distributions
- hybrid & improved MCMC methods
- *Reliable* assessment of out-of-sample performance

- `http://ms.mcmaster.ca/~bolker/misc/private/14-Fox-Chap13.pdf`
- `https://rawgit.com/bbolker/mixedmodels-misc/master/ecostats_chap.html`
- (B. M. Bolker 2015)

  (code ASPROMP8)

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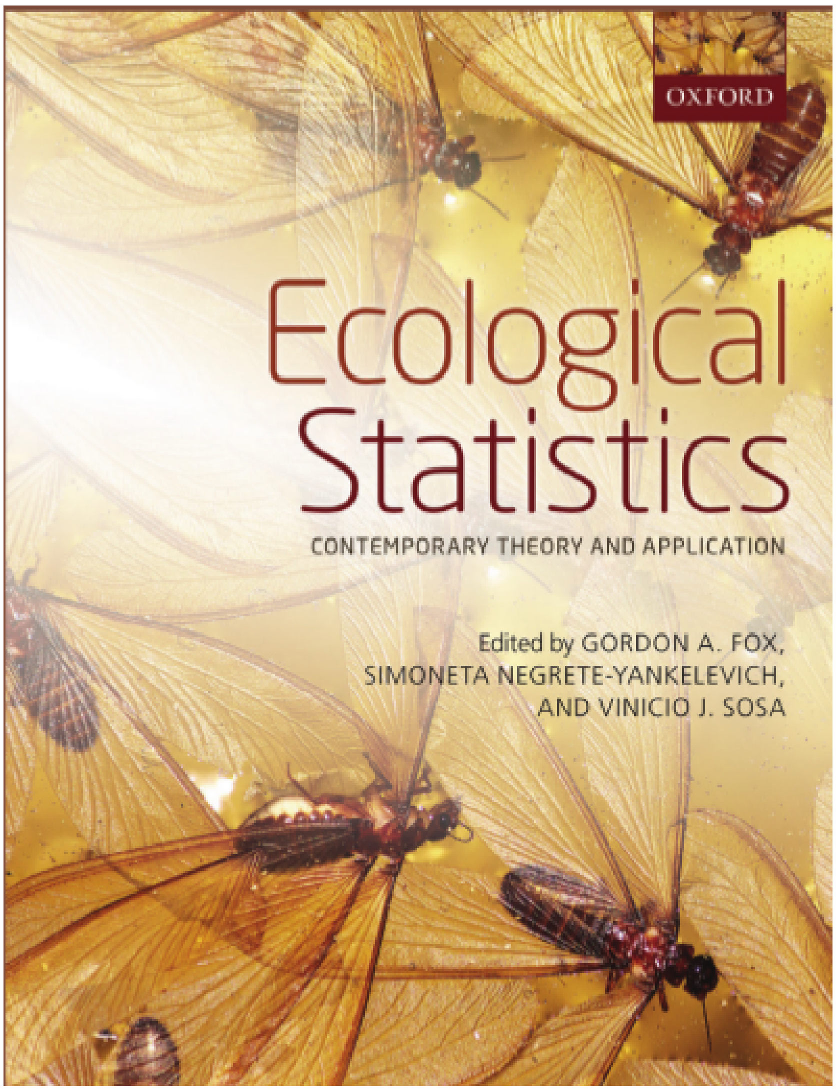

Figure 3: image

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