

Discrete-time Lotka-Volterra model:

$$\begin{aligned}V_{t+1} &= rV_t - aP_tV_t \\ P_{t+1} &= sP_t + acP_tV_t\end{aligned}$$

$V$  is prey (“victim”),  $P$  is predator.  $r$  is prey growth rate;  $a$  is attack rate of predators;  $c$  is conversion efficiency of predators;  $s$  is predator survival in the absence of prey.

Assume  $\{r, a, c\} > 0$ ,  $0 < s < 1$ .

Compute equilibria:  $\{0, 0\}$ ,  $\{(1-s)/(ac), (r-1)/a\}$

Calculate Jacobian:

$$\begin{pmatrix} r - aP^* & -aV^* \\ acP^* & s + acV^* \end{pmatrix}$$

Stability of 0 eq:  $J$  is

$$\begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}$$

We hardly need the Jury conditions  $|T| < 1 + \Delta < 2$  for this case, we can read off the diagonals. Zero equilibrium is stable iff  $r < 0$  (we have  $0 < s < 1$  by assumption).

Jacobian at non-trivial equilibrium:

$$\begin{pmatrix} 1 & (s-1)/c \\ (r-1)c & 1 \end{pmatrix}$$

$T = 2$ ;  $\Delta = 1 - (r-1)(s-1)$ . We need  $2 < 1 + (r-1)(s-1) < 2$ . This can never be stable.