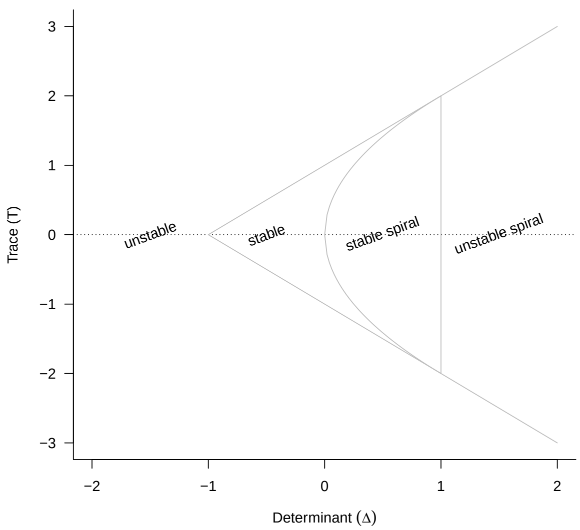
2D nonlinear discrete-time models

Nicholson-Bailey

Equations: $V_{t+1} = rV_t e^{-qP_t}$, $P_{t+1} = cV_t \left(1 - e^{-qP_t}\right)$.

Equilibria: $\{0, 0\}$ and $\{r \log r/(qc(r-1), \log r/q\}$. P^* increases with r and decreases with q (more effective $P \rightarrow$ *fewer* P). V^* decreases with c and q (not surprising): increases with r (denom. of derivative = $r - 1 - logr) > 0$ for $r > 1$).

Stability: need *Jury conditions*: $|T| < 1 + \Delta < 2$. (These are shortcuts



(adapted from (1))
For N-B, Jacobian is

$$\begin{pmatrix} re^{-qP^*} & -rqV^*e^{-qP^*} \\ c\left(1 - e^{-qP^*}\right) & qcV^*e^{-qP^*} \end{pmatrix}$$

At the trivial equilibrium this is $\begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix}$; so $T = r$, $\Delta = 0$. Boring.

At the positive equilibrium we get $T = 1 + (\log r/(r-1)$, $\Delta = (r \log r)/(r-1)$. Can show this is > 1 when $r > 1$ ... therefore we are definitely unstable (exercise: determine numerically when $\Delta > T$ and vice versa (i.e. unstable spirals vs. real instability). Examine the behavior of the N-B model starting from near the equilibrium to confirm your finding.)

How does this match reality (see chapter 4 of (1) for more detail)? What can we do about it?

References

[1] S. P. Ellner and J. Guckenheimer. *Dynamic Models in Biology*. Princeton University Press, Princeton, NJ, 2006.