

4 Mar 2021

BOUNDED

- sensitive dependence on initial conditions

unstable
if



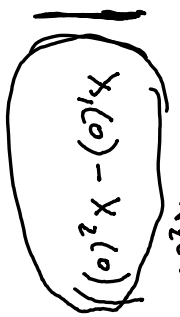
$$\frac{dx}{dt} = rx \quad x \geq 0$$

$r > 0$

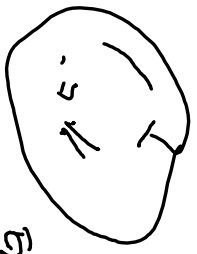
'quickly':

Lyapunov exponent

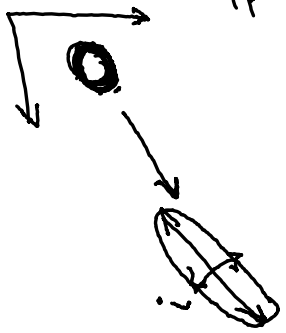
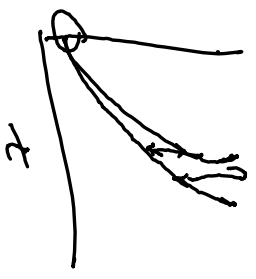
$$\Delta(\text{tra}_j, \text{tra}_k) \sim e^{\lambda t}$$



continuous-time
deterministic



EXPANSION or
CONTRACTION of
a volume of phase space



SPECTRUM

$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \dots$

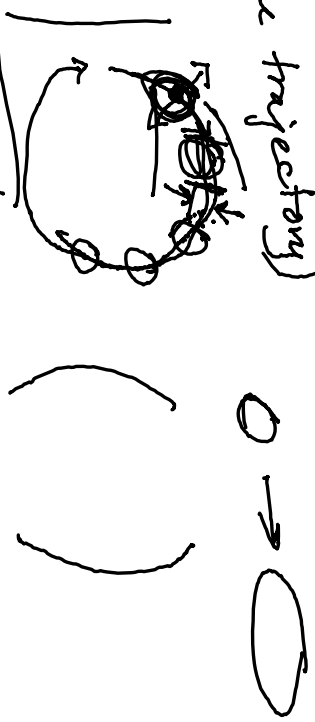
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$\lambda_1, \lambda_2, \dots$

average eigenvalues of the system
(averaged over the trajectory)

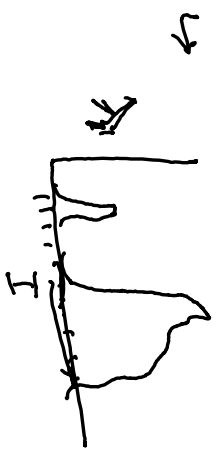
$$J(t) = \lambda \dots$$

$$\left(\begin{matrix} J_1 & & & \\ & J_2 & & \\ & & J_3 & \\ & & & J_4 \end{matrix} \right) \approx \text{numerically impossible to integrate analytically?}$$



numerically integrate
Gram-Schmidt orthonormalization
Lyapunov exponent

Lyapunov spectrum



$\lambda_0, \lambda_2, \lambda_3 \dots$

contracting

expanding



DIMENSIONALITY: Chaotic attractors are often FRACTAL

(non-integer dimensions) Δ
infinitely self-similar

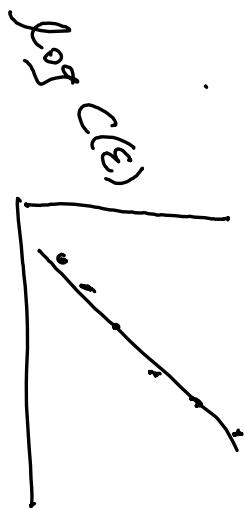
DIMENSIONALITY:

correlation: number of points within distance ϵ

$$C(\epsilon) \propto \epsilon^D$$



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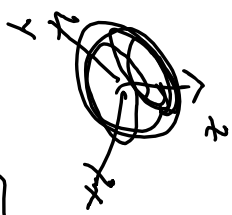


$$\log(y) = a + b \log(x)$$

$$y = e^a \cdot e^{b \log x} =$$

$$e^a \cdot (e^{\log x})^b = e^a \cdot x^b$$

POWER-LAW $C(\epsilon) \propto \epsilon^b$



$2.2 \pm 0.1 \rightarrow$ not an integer!

\rightarrow CHAOS (or quasiperiodicity)

EGG trace } $\int \dots \int \dots \int \dots$

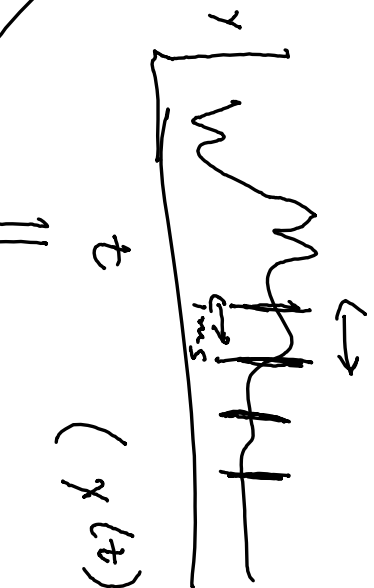
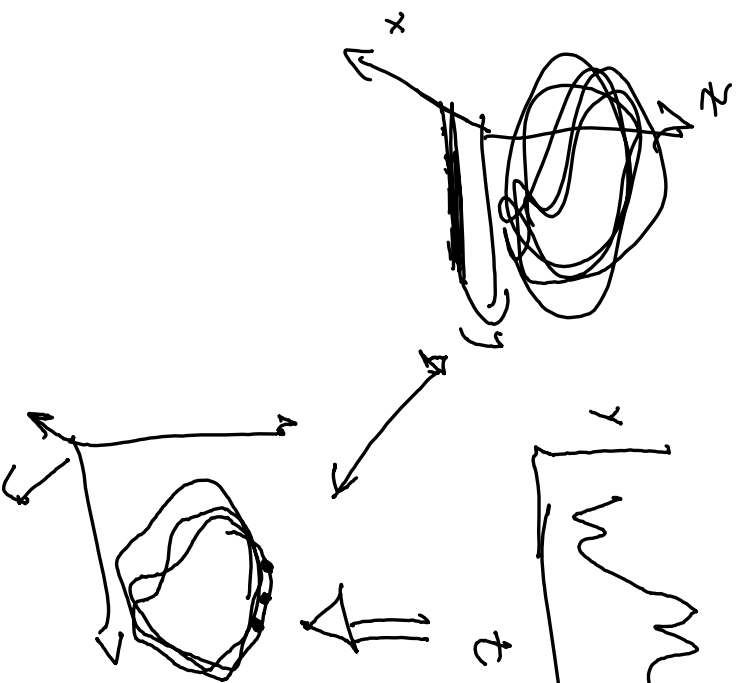
chaotic??

TAKENS theorem

single state variable from a complicated system

high dim trajectory

\rightarrow 1 state variable \rightarrow embed in a higher dimension \rightarrow same TOPOLOGY



$(y(t), y(t+\tau), \dots)$
 $y(t+k\tau)$

$k = \text{embedding lag dimension}$
 $\tau = \text{embedding lag}$

\approx FRACTAL dimension

Kaplan-Yorke dimension
 - Lyapunov spectrum

