LOGISTICS

8 Feb 2021

HW 2
- group assignment
- due on Friday
- FORM GROUPS ASAP.
  email or Zulip.
- project ideas: end of reading week
- 2-4 people.

Greek letters in code: γ, β, ϕ : not portable
beta, gamma, phi

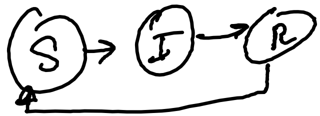
- THIS WEEK. - epidemic models.
  stochasticity, spatial, evolution

ENDEMIC DISEASE: endemic equil.

8 Feb p 2

SIR model: SINGLE EPIDEMIC.

SIRS model. (waning immunity)



flu (evolution)

SARS-CoV-2 (???)

(coronaviruses)

cholera (King et al)

"slow diseases":

HIV, tuberculosis,

(animals + plants)

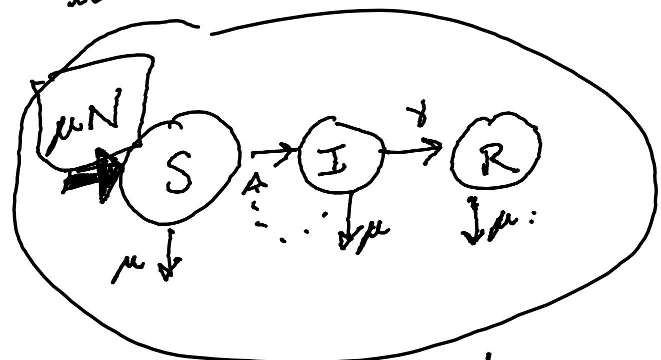
$\mu \approx \frac{1}{70}\,\mathrm{yr}^{-1}$

SIR model with vital dynamics

- new people are being born ($\to S$)
  - vertical transmission ($\to I$?)
  - maternal immunity ($\to R$!)

CONSTANT POPULATION

(growing pop?) DEMOGRAPHIC vs EPIDEMIC



$$\frac{d(S+I+R)}{dt} = \mu N - \mu S - \mu I - \mu R = 0$$

$$\frac{dS}{dt} = \mu N - \frac{\beta}{N} SI - \mu S$$

$$\frac{dI}{dt} \quad \frac{\beta}{N} SI - (\mu + \gamma) I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

scaling by N

- YES: data, real pop'ns.
- NO: math:

$S' \to \frac{S}{N}$, $I' \to \frac{I}{N}$, $R' \to \frac{R}{N}$

proportions.

$$S^* = \frac{\mu + \gamma}{\beta} = \frac{1}{R_0} \quad ***$$

$$I^* = \frac{\mu}{\beta}(R_0 - 1)$$

everybody is born susceptible.

FORCE OF INFECTION (per cap rate of inf of susc):

$\Lambda = (\beta I)$: FRACTION of people uninf at age a = $e^{-(\beta I^*)a}$

exp distrib AGE AT INFECTION

average age at first inf $= \dfrac{1}{\beta I^*} = \dfrac{1}{\cancel{\beta}\left(\frac{\mu}{\cancel{\beta}}(R_0 - 1)\right)}$

average lifespan $= \dfrac{1}{\mu}$

$$\frac{L}{A} = (R_0 - 1) \rightarrow \frac{A}{L} = \frac{1}{R_0 - 1} =$$

measles. $R_0 \simeq 15$. $\dfrac{1}{14} \sim \dfrac{70}{14} \sim 5$ years old.

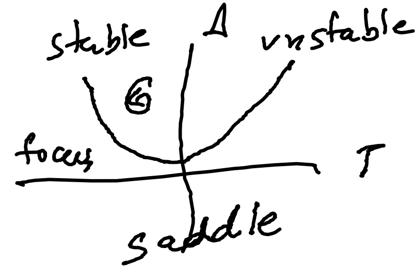
I know A, how do I find R_0?

$$R_0 - 1 = \frac{L}{A} \approx \frac{L}{20} \approx \frac{70}{20} = 3.5 \rightarrow R_0 = 4.5$$

$\boxed{S^* = \frac{1}{R_0}} \Rightarrow$ serosurvey: R^* : $S^* \approx 1 - R^*$

• HW, so far:

SIGN OF REAL PART of dom eigenvalue

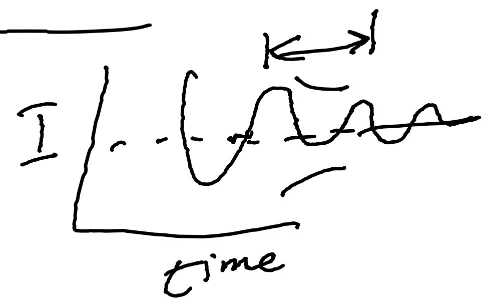


$$\lambda = \underbrace{\text{re part}}_{A} + \underbrace{\text{im part}}_{B}$$

$$e^{At} \cdot e^{iBt} = e^{At} \cdot (\cos Bt + i \sin Bt)$$

$$\text{period} = \frac{2\pi}{B}.$$

$$\begin{pmatrix} -\mu R_0 & -(\mu+\gamma) \\ \mu(R_0-1) & 0 \end{pmatrix}$$



$$\lambda = \frac{1}{2}\left(-\mu R_0 \pm \underbrace{\sqrt{\mu^2 R_0^2 - 4\mu(R_0-1)(\mu+\gamma)}}\right)$$

$\mu \ll \gamma$ (demography is slower than the epidemic)

$A \gg \frac{1}{\gamma}$

↓ ASSUME spiral

$$\lambda = \left(\frac{1}{2}x - \mu R_0\right) \pm i\sqrt{\gamma\mu(R_0-1)}$$

$$= \frac{-\mu R_0}{2} \pm i\sqrt{\gamma / A}$$

A = AGE AT FIRST INFECTION

PERIOD $\propto \underbrace{\sqrt{A/\gamma}}$ $\Rightarrow$ COVID??? $R_0 \approx 3$:

$A(R_0)$, L (host biology) ; $\frac{1}{\gamma}$ (epidemic SPEED)

R_0 (unitless)