8 Mar 2021

- LYAPUNOV exponents

(know the equations)

$J(x_1, y_1, z_1)$

$J_1 J_2 \cdots J_n$

$$\begin{pmatrix} \frac{\partial g_i}{\partial x_j} \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} :$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} : \text{NEUTRAL} \qquad \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \\ & & \lambda_3 \end{pmatrix} \quad \lambda_1 > 0$$

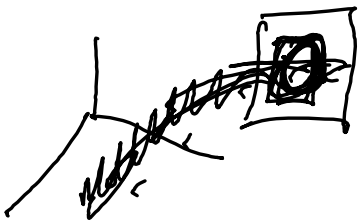
expansion in all directions: ALL-positive Lyap spectrum

finding chaos in equations vs
finding chaos in real data.

- EQUATIONS: find long-term average eigenvalues.
- sample of points: (time series).
  IF knew all of the state variables:
  correlation dimension.
  (how does the avg # of points within distance d of a point scale with d? $\log \left| \frac{}{\log d} \right.$) ASK: is d non-integer?
- only have one state variable:
  EMBEDDING ~ into as many dimensions as we like
  state $\{X_t, X_{t+\tau}, X_{t+2\tau}, \cdots\}$

PROBLEM with the dimension- detectors

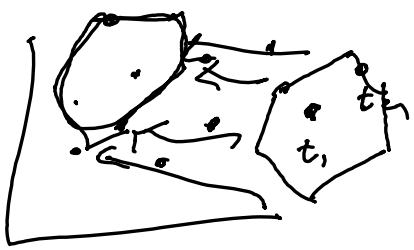
(corr. dimension): sensitive to noise



. would like to follow trajectories

RECONSTRUCT trajectories

RECONSTRUCT equations.



$$\Delta x_{pred} = \beta_0 + \beta_1 P + \beta_2 V + \beta_3 PV + \beta_4 P^2 + \beta_5 V^2$$

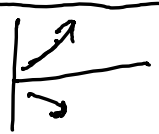
$$\Delta p_{prey}(V) =$$

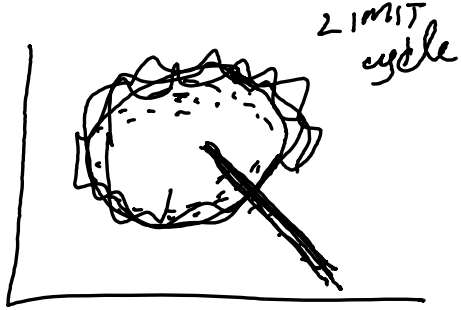
Is Mother Nature a chaotic attractor?

(NOT? the most interesting question).

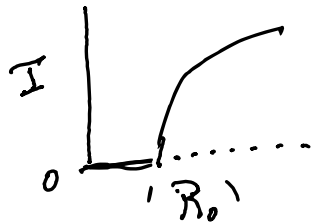
$\lambda = 0.001\ year^{-1}$

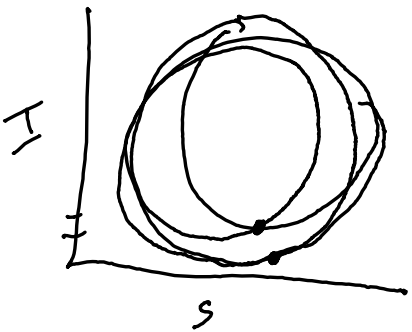
$\lambda = -0.001\ year^{-1}$





NUMERICAL BIFURCATION ANALYSIS





- run the model for many values of param
- discard a transient
- record the behaviour
  - min, max
  - POINCARÉ section
  - stroboscopic MAP (periodicity?)