PRECISE / REALITY / GENERAL.

JAN 20 p1

(Levins 1966)

$y = f(x), f'(x) > 0$ ← × precise

SIR equations : PARAMETERS
FUNCTIONAL FORMS

- $y = a + bx$ ← generality, precision × realism
- $y = 2.37 + 4.65x$ ← precision, realism × general-

precision = can we make QUANTITATIVE predictions?

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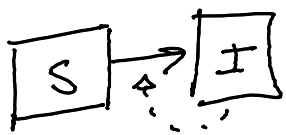
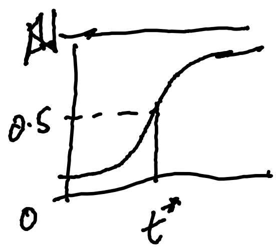
simple epidemic



$$\frac{dS}{dt} = -\beta S I \qquad \frac{dI}{dt} = \beta S I$$



$$\Rightarrow \frac{dI}{dt} = \beta (N - I) I$$

SOLVE (by partial fractions)

$$\approx \frac{1}{1 + e^{-\beta(t - t^*)}}$$

I(0)?

LOGISTIC

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right):$$

NONDIMENSIONALIZATION

Jan 20 p 2

what parameters can we 'get rid of' without loss of generality ??

can I set some params to a 'special' value (usually 1) without changing the QUALITATIVE range of behaviours of the model ?

· exponential growth : if I assume $x(0) > 0$ &

$$x = x(0)\, e^{rt} :$$

change vars to $\boxed{x' = \dfrac{x}{x(0)}}$ $\rightarrow$ $x' = e^{rt}$

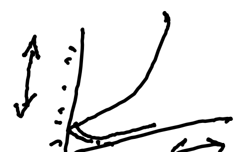
$x(0) \rightarrow 1$

IF $r > 0$

$x' = e^{t}$

$(t' = rt)$

$x_0 = 1$



$r = 1$

for every DISTINCT unit in the model I usually have 1 'degree of freedom' / 1 parameter that I can get rid of.

$$\frac{dI}{dt} = \underbrace{(\beta - \gamma)}_{r}\, I \qquad \text{(no susceptible depletion)}$$

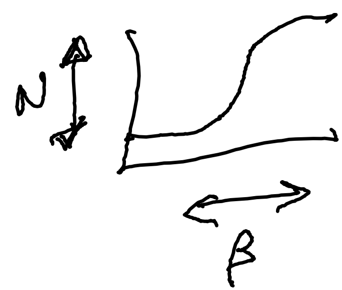
$r \to 1$

$I(0) \to 1$

$$\frac{dI}{dt} = \beta (N - I) \cdot I \; :$$

$N \to 1$ (pop'n/pop density → prop.)

$\beta? \to 1$ (timescale)



<u>ADVANTAGES</u> of non-dim.

- easier algebra!
- generality ('canonical form')

<u>DISADVANTAGES.</u>

- less precise → compare with reality
- makes it harder to do unit checks

LOGISTIC equation

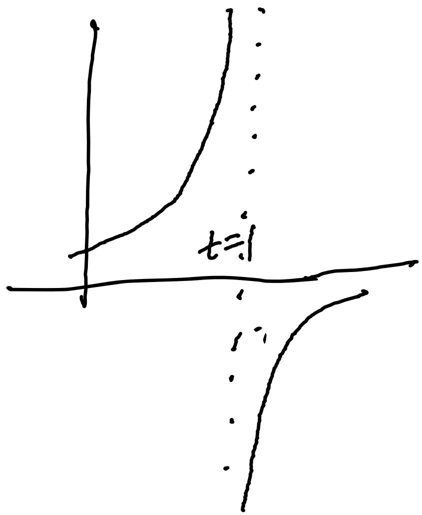
prec. vs gen. vs realism (Levins 1966)

- is it biologically well posed?
  (if starting vals & parameters all have sensible signs do the solutions always end up sensible?)

(all pops start ≥ 0) → stay ≥ 0 forever?
(want solutions to stay BOUNDED in finite time)

If we had a model where $x(t) = \frac{x(0)}{1-t}$ and we started at time 0 . . .

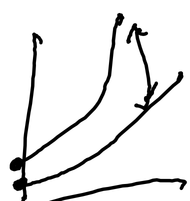
$x(0) > 0$



- well-posed ?
- ATTRACTORS
  - point (at infinity)
  - orbit (limit cycle)
  - chaotic attractor
  - quasiperiodicity

bounded : positive Lyapunov exponent
(sensitive dep on ICs)
(nearby trajectories div exponentially)

$(x_1(0) - x_2(0))$
e^{rt}



unbounded