1. biologically well posed
(state variables remain ≥ 0
if ICs are non-negative)

2. characterize attractors
(equilibria)
(point, limit cycles, chaos, QP...)

~ existence? → general, realistic
qualitative.
~ locate them
precise?

~ STABILITY (& biological realistic?)
~ local asymptotic ~ multiple. REGION
global asymptotic → for some region
↳ infinitesimal

Lyapunov f'n:
$f(x) \geq 0$
$= 0$ only at equil
$f'(x) < 0$ but at equil

('like integration'
= HARD

linearization → 'like differentiation'

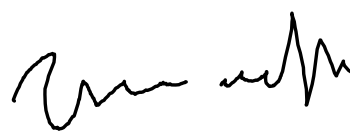
21 Jan p 2

LINEARIZATION

- find the Jacobian
  (derivs of the gradients)
  N x N matrix $\left(\frac{\partial g_i}{\partial x_j} \right)$
- evaluate J. at the equilibrium
- evaluate the eigenvalues
  PRINCIPAL e'val (λ max)

> 0 → unstable
< 0 – stable

if it has imaginary parts
~ spiral



BIFURCATION analysis:

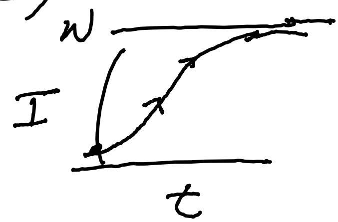
parameter space

$\beta > 0$ unstable
$\beta < 0$ stable
$\beta = 0$ bifurcation point

LOGISTIC (simple epidemic)

$$\frac{dI}{dt} = \boxed{\beta I(N - I)} \quad \frac{\partial}{\partial I}$$

Jacobian: $(\beta(N-2I))$



$I^* = 0 \quad [\beta N]$

$I^* = N \quad [-\beta N]$

$N > 0$: if $\beta > 0$ disease-free equilibrium (DFE) is unstable

ENDEMIC eq is stable

$\beta < 0$ then it's reversed

$\beta = 0$ bifurcation point

$$\frac{dI}{dt} = I\,(f(I))$$

$f(I) \geq 0$
$0 < I < 1$

$f(I) = 0$
only at $I = N$

SIR model

$$S \to I \to R$$

$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I - \gamma I$$

recovery rate
removal rate

$$\frac{dR}{dt} = +\gamma I$$

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \rightarrow \frac{d(S+I+R)}{dt} = 0$$

pop size fixed
(as long we count R)

looks 3D, but we only need 2

normally we would substitute

$$R = N - S - I \leftarrow$$

UNITS: β : per infected · per time

$\gamma : t^{-1} \rightarrow \frac{1}{\gamma}$ MEAN infectious period

$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I - \gamma I$$

$\beta > 0$

PHASE PLANE

<u>null clines</u>

$\left(\frac{dx_i}{dt} = 0\right)$: both axes for both S and I

$\overline{S = 0}$ $\frac{dS}{dt} = 0$

can't cross axes

↪ system is biologically well posed

$S = 0$

<u>EQUILIBRIA</u>

DFE: entire S axis

($I=0$) any $\{S, R\}$

$S + R = 1$ ($I = 0$)