22 February

LOGISTICS

- feedback on proj proposals (*) Feb 24
- form / questionnaire.

ONE MORE DAY on epidemic models.

- GILLESPIE algorithm.
  cont time, discrete state, stoch models
  competing Poisson processes. pick Exp(rate = Σ rates)
  - U[0,1] $\rightsquigarrow$ which event?

SUPER SLOW. Scales with pop size.

ODE ~ constant in pop size
R-F ~ constant · Binom(,)

(TAU-LEAPING) $\approx$ Poisson / binomial approximations

Negative Binomial

Bernoulli 'coin-flipping' process
how many trials before k successes?

overdispersed Poisson

$\lambda_i \sim \text{Gamma}(\text{mean}, \text{shape})$

$X_i \sim \text{Poisson}(\lambda_i) \rightarrow X_i \sim NB(\mu = \lambda_i, \text{disp} = \text{shape})$

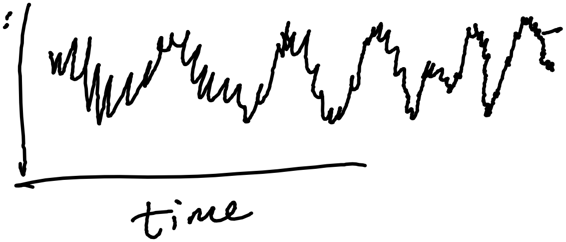
(Ali's question): tau-leaping. choose NB vs Poisson?

- stochastic ODEs: continuous time, stochastic, continuous state

  Poisson process → CENTRAL LIMIT theorem → changes in pop size will be Gaussian.

continuous-time, continuous-state stoch processes.

deriv. <u>WIENER process</u>:



CONTINUOUS.
NOWHERE differentiable

$$\frac{dX}{dt} = \underbrace{f(X(t))}_{\text{determ}} + \underbrace{\sigma(X(t))\, dW(t)}_{\text{stoch.}}$$

$$C = E\begin{pmatrix} (\Delta S)^2 & (\Delta S \Delta I) \\ \Delta S \Delta I & (\Delta I)^2 \end{pmatrix} = \text{rate} \cdot (\underset{(\pm 1)}{\text{Change in state}})^2$$

$$= \begin{pmatrix} \beta SI/N & -\beta SI/N \\ -\beta \frac{SI}{N} & \beta \frac{SI}{N} + \gamma I \end{pmatrix}$$

$$G G^T = C$$ MATRIX SQUARE ROOT

$$\begin{pmatrix} -\sqrt{\beta S I / N} & 0 \\ \sqrt{\beta S I / N} & -\sqrt{\gamma I} \end{pmatrix}$$

CHOLESKY decomposition

SIMULATE the process.

Euler-Maruyama.

$$X(t+\Delta t) = \underbrace{X(t) + f(x(t)) \cdot \Delta t}_{\text{Euler}} + \underbrace{G \cdot \eta \cdot \sqrt{\Delta t}}_{\text{stoch.}}$$

stoch. → DIFFUSION process

$$\eta = \text{vector of } N(0,1):$$

var of stoch term:

$$\left(G \eta \sqrt{\Delta t}\right)\left(G \eta \sqrt{\Delta t}\right)^T = C \Delta t$$

$$\left(G G^T\right)\underbrace{\left(\eta \cdot \eta^T\right)}_{\sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \cdot \Delta t$$