

• well-posed? (math + biological)

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Solns exist and are unique

biologically 'impossible' dynamics don't happen

(Solutions are non-negative and? bounded?)

• attractors (equilibria)

• local/global? stability (Lyap functions)

↳ Jacobian, linearization, etc.

• bifurcation structure (when does stab of attractors change?)

\* PHASE PLANE

plot eq, nullclines, flow field > trajectories



$$grad = g_x, g_y, g_z$$

$$g_x(x(t), y(t), z(t), t)$$

dep<sup>on</sup> non autonomous  
t

FORCED

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tools for phase plane analysis:

XP PART

phase R

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# BIFURCATION analysis

changes in stability

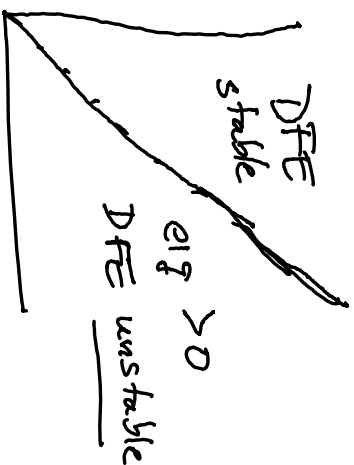
bifurcation parameter

eigenvalue =  $\beta - \gamma$

DFE changes stab when  $\beta = \gamma$



$\gamma$



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$$R_0 = \left[ \frac{\beta}{\gamma} \right]$$

intrinsic (basic) reproductive number

units of  $R_0$ ?

UNITLESS : : 1 /  $\gamma$  infectious period

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\gamma \sim \left[ \frac{1}{t} \right]^{-1}$$

$$\beta \sim \left[ \frac{1}{t} \right]^{-1}$$

meaning =  $\beta SI \sim \beta I$

$$I(0) = 1 : \beta$$

$$\gamma = 1 \text{ week}^{-1}$$

$$\frac{1}{\gamma} = 1 \text{ week}^{-1}$$

$$\beta = 3 \text{ week}^{-1}$$

$R_0 > 1$  DFE unstable

$R_0 < 1$  DFE stable

rate of change  
per disease  
generation

$$R_0 \geq 1$$

$$\frac{\beta}{\gamma} = 3$$

EXPECTED number of new  
cases from 1 infected case  
in an otherwise susceptible pop  
(near the DFE)

$$r = \beta - \gamma$$

rate of change  
per time

$$\frac{\log(2)}{r}$$

$$r \leq 0 ?$$

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