

Brief stuff on deterministic chaos

Ben Bolker

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Chaos

- sensitive dependence on initial conditions (sdic); nearby trajectories diverge (exponentially)
- bounded

(exponential growth is an example of **unbounded** sdic)

Lyapunov exponent/spectrum

- Long-term average dynamics
- Expansion/contraction of phase space

```
par(las=1,bty="l")
library(ellipse)
ee <- ellipse(x=0,t=1)
M <- matrix(c(1.2,0.5,0.5,1),nrow=2)
MASS::eqsplot(ee,type="l",xlim=c(-5,5),axes=FALSE)
box()
for (i in 2:5) {
  ee <- t(M %*% t(ee))
  lines(ee[,1],ee[,2],col=i)
}
```

Fractal dimension

- Kaplan-Yorke:
- Correlation dimension (Grassberger-Procaccia)
 - $C(\epsilon) = \lim_{N \rightarrow \infty} g/N^2$
 - $C(\epsilon) \propto \epsilon^{\nu}$
 - log-log graph
- box-counting

Embedding

- Takens theorem

Poincaré map

References

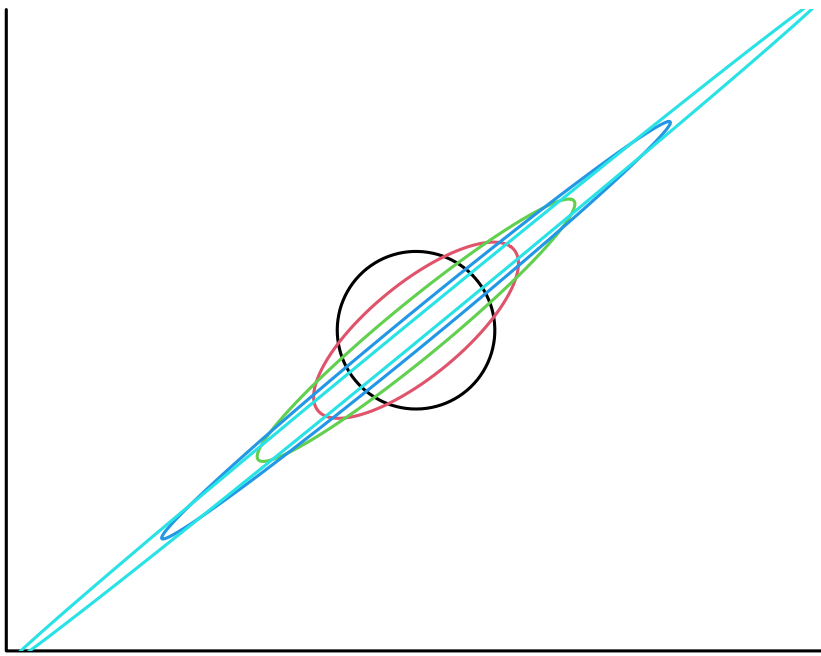


Figure 1: scaling by repeated transformation